

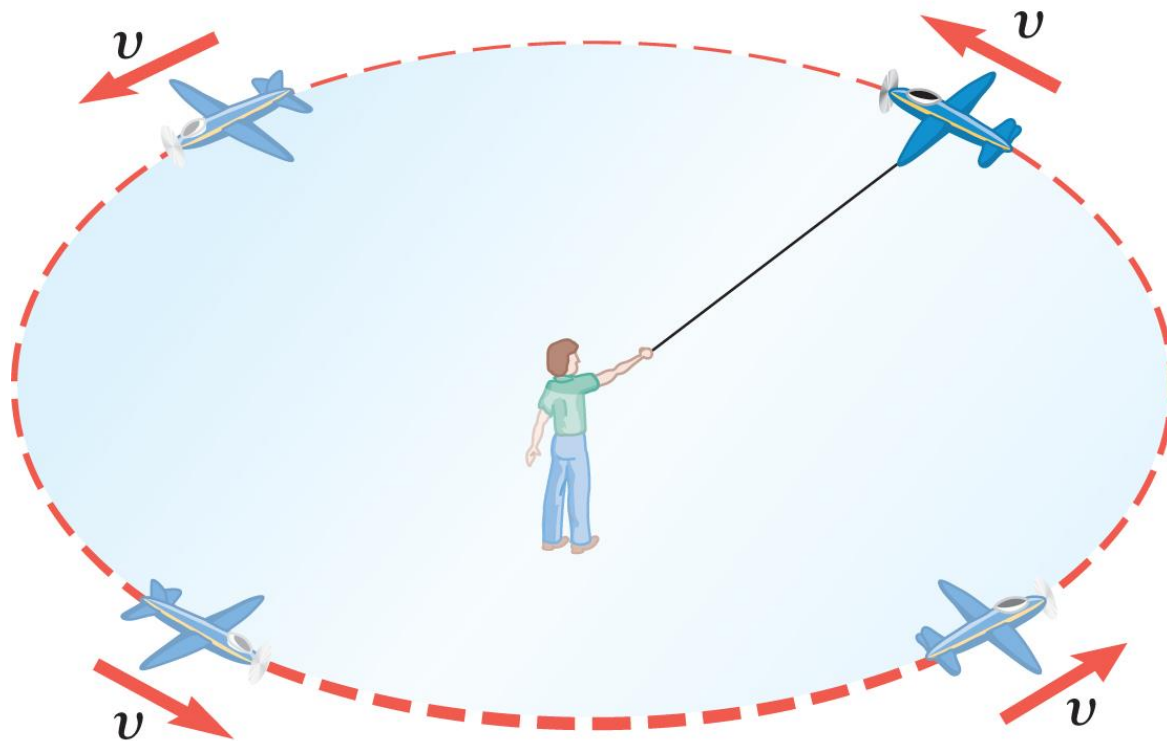
Chapter 5

Dynamics of Uniform Circular Motion

5.1 Uniform Circular Motion

DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



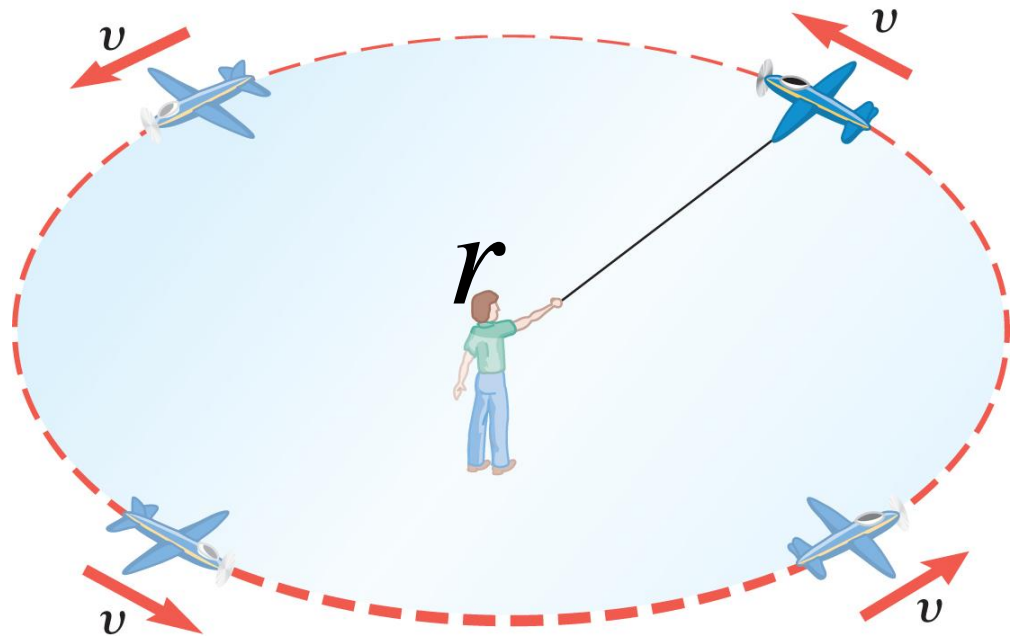
5.1 Uniform Circular Motion

Let T be the time it takes for the object to travel once around the circle.

Distance travelled in one circle is? $2\pi r$

$$v = \frac{\text{distance travelled}}{\text{time elapsed}}$$

$$v = \frac{2\pi r}{T}$$



Example 1: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

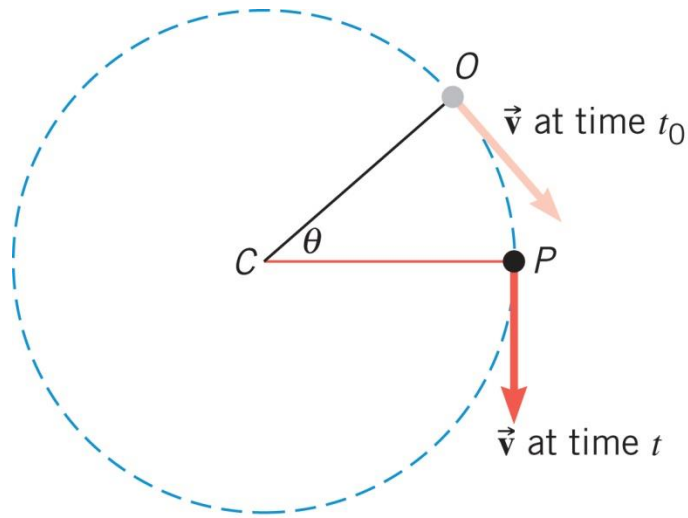
$$\frac{1}{830 \text{ revolutions/min}} = 1.2 \times 10^{-3} \text{ min/revolution}$$

$$T = 1.2 \times 10^{-3} \text{ min} = 0.072 \text{ s}$$

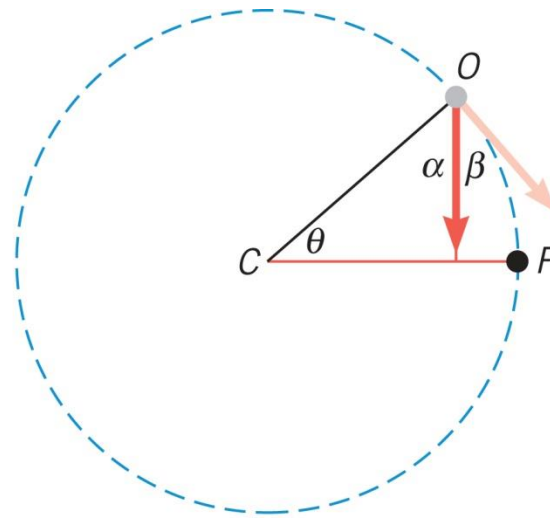
$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29 \text{ m})}{0.072 \text{ s}} = 25 \text{ m/s}$$

5.2 Centripetal Acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant*.



(a)



(b)

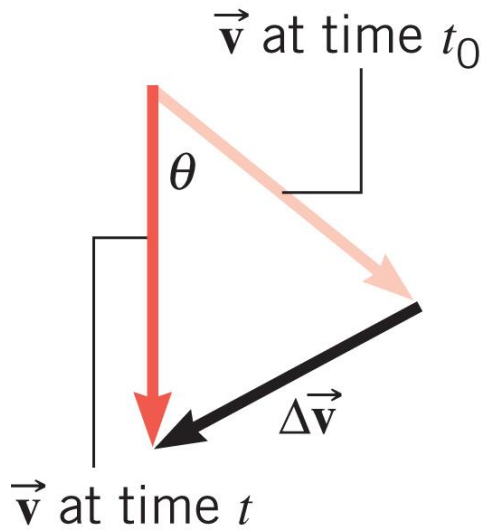
$$\alpha + \beta = 90^\circ$$

$$\alpha + \theta = 90^\circ$$

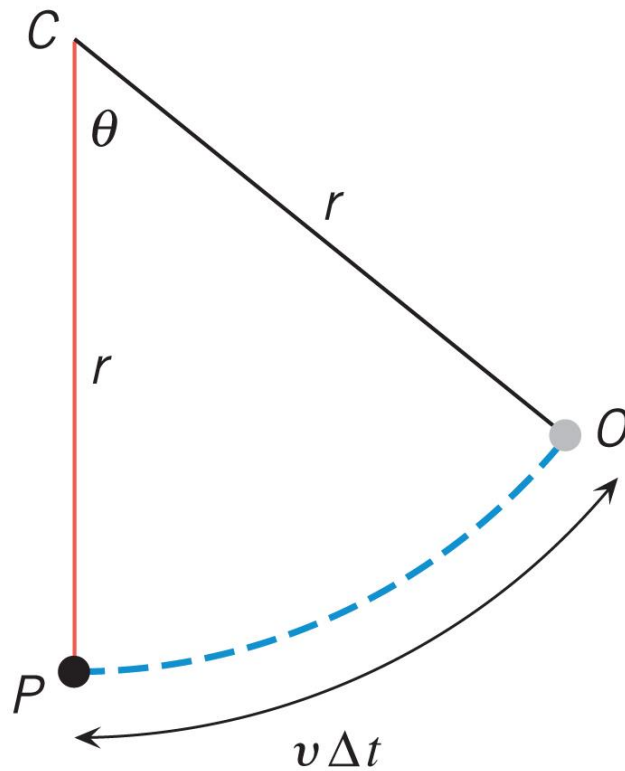


$$\beta = \theta$$

5.2 Centripetal Acceleration



(a)



(b)

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$



$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

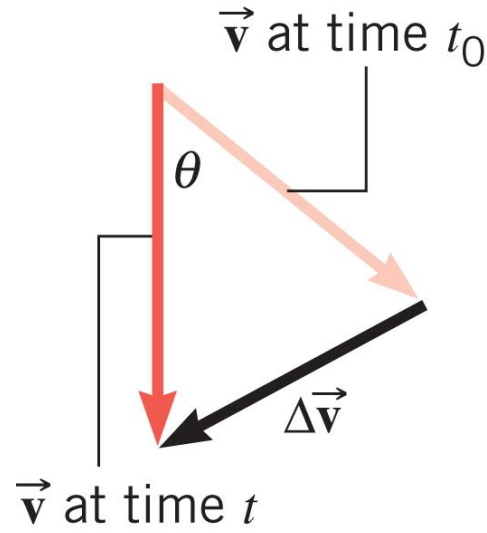


$$a_c = \frac{v^2}{r}$$

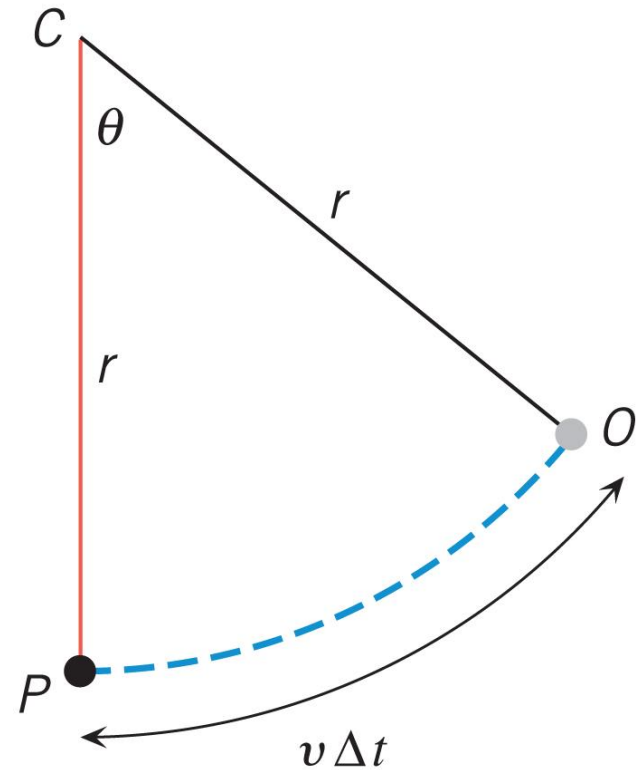
5.2 Centripetal Acceleration

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.

$$a_c = \frac{v^2}{r}$$



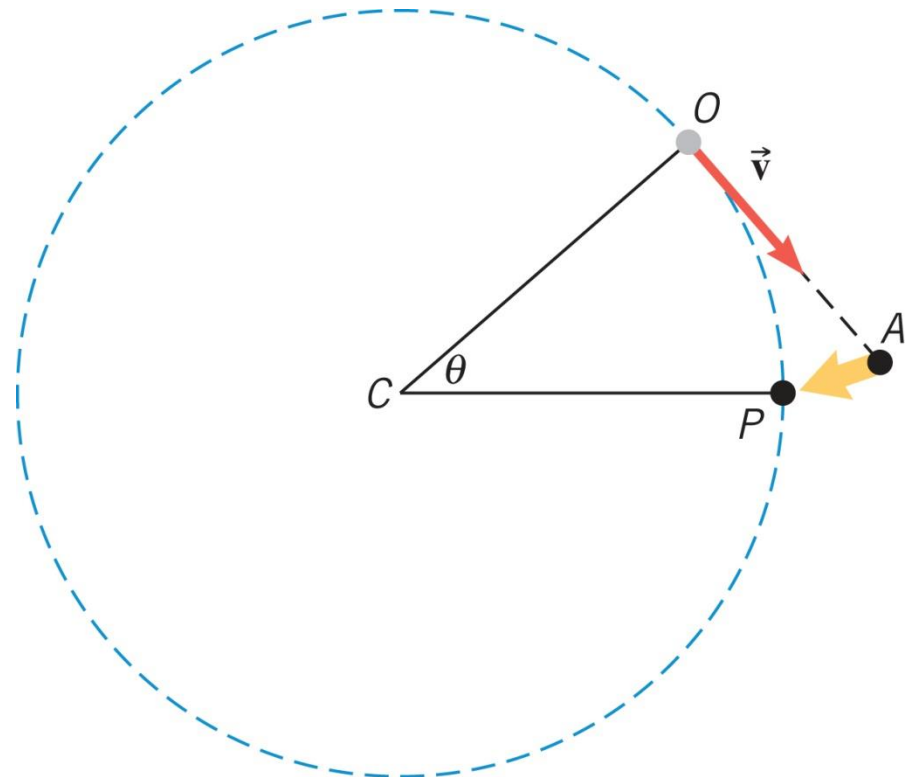
(a)



(b)

Conceptual Example 2: Which Way Will the Object Go?

An object is in uniform circular motion. At point O it is released from its circular path. Does the object move along the straight path between O and A or along the circular arc between points O and P ?



Recall Newton's Second Law

When a net external force acts on an object of mass m , the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

5.3 Centripetal Force

Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

The centripetal force is the name given to the net force required to keep an object moving on a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

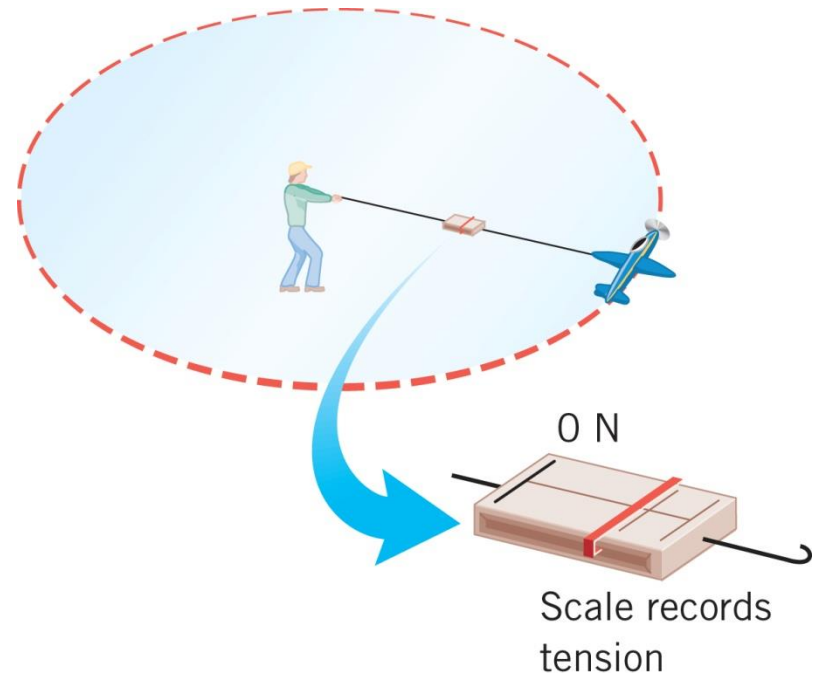
$$F_c = ma_c = m \frac{v^2}{r}$$

Example 5: The Effect of Speed on Centripetal Force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

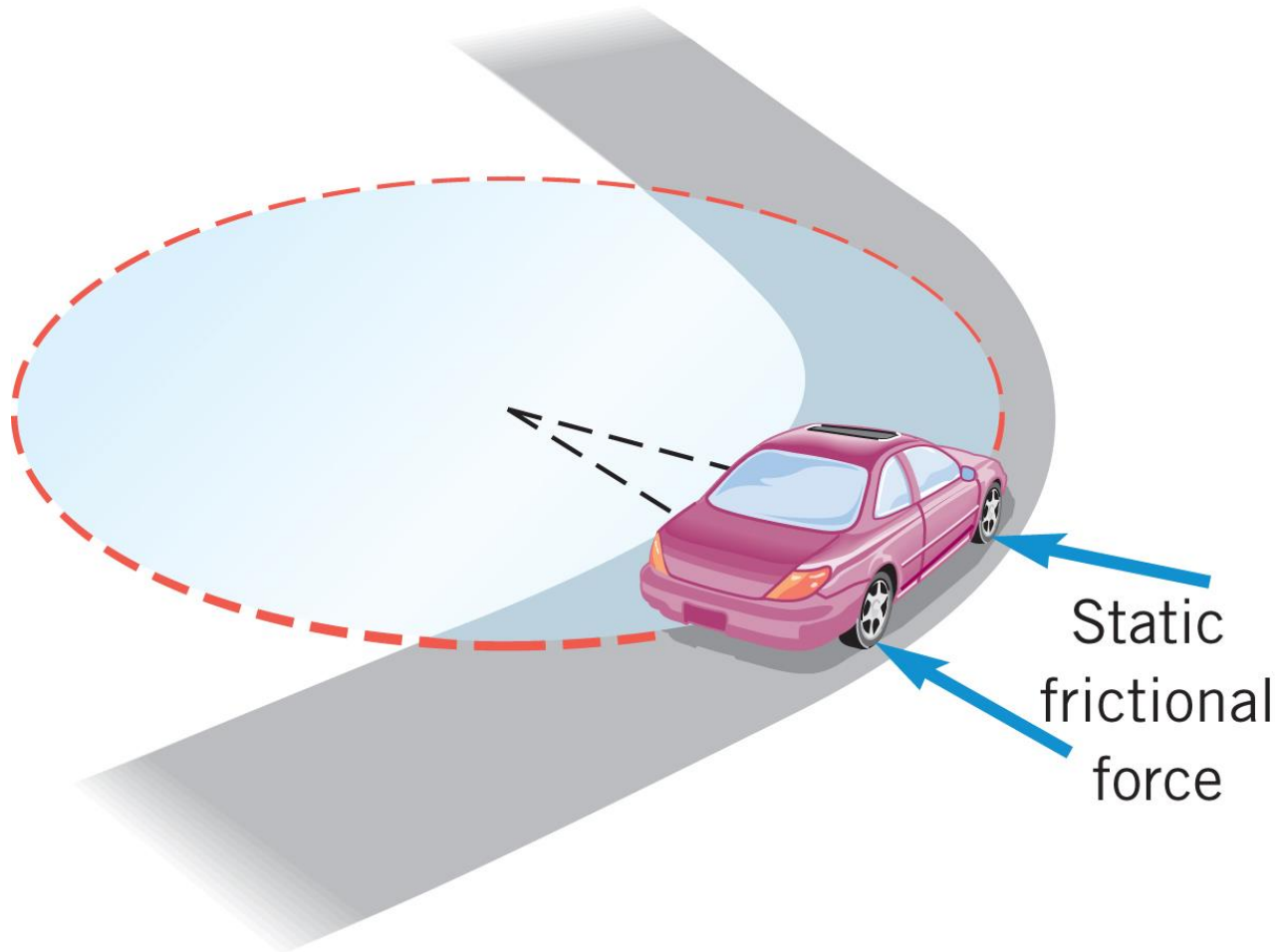
$$F_c = m \frac{v^2}{r}$$

19.1 N



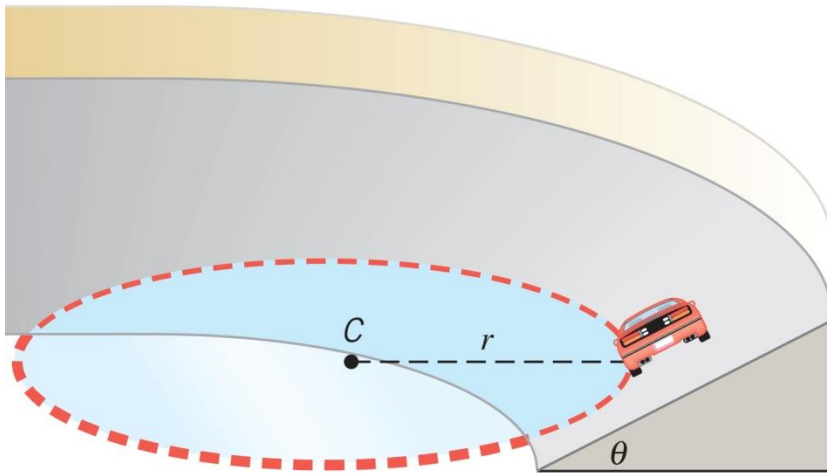
5.4 Banked Curves

On an unbanked curve, the static frictional force provides the centripetal force.

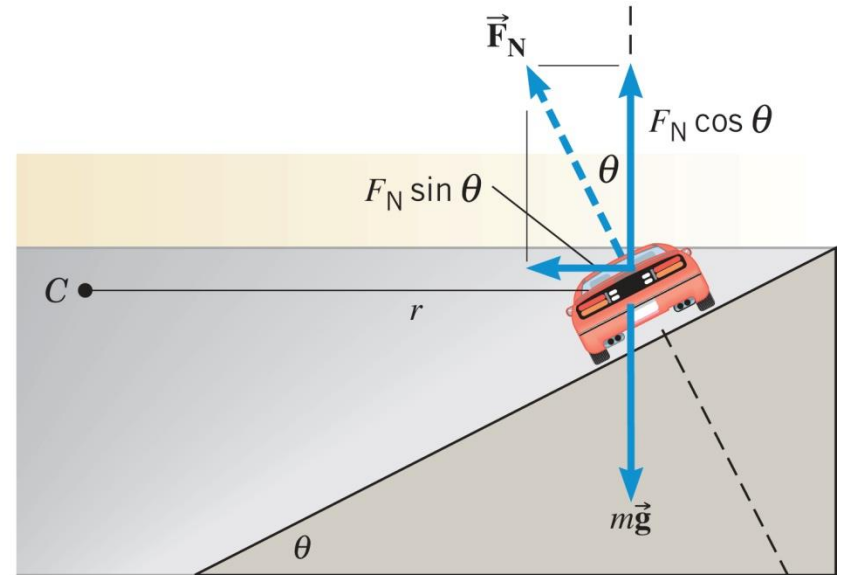


5.4 Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.

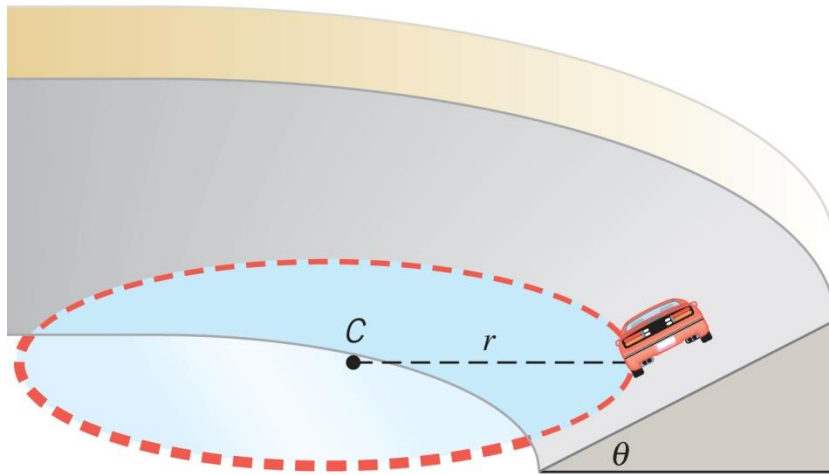


(a)

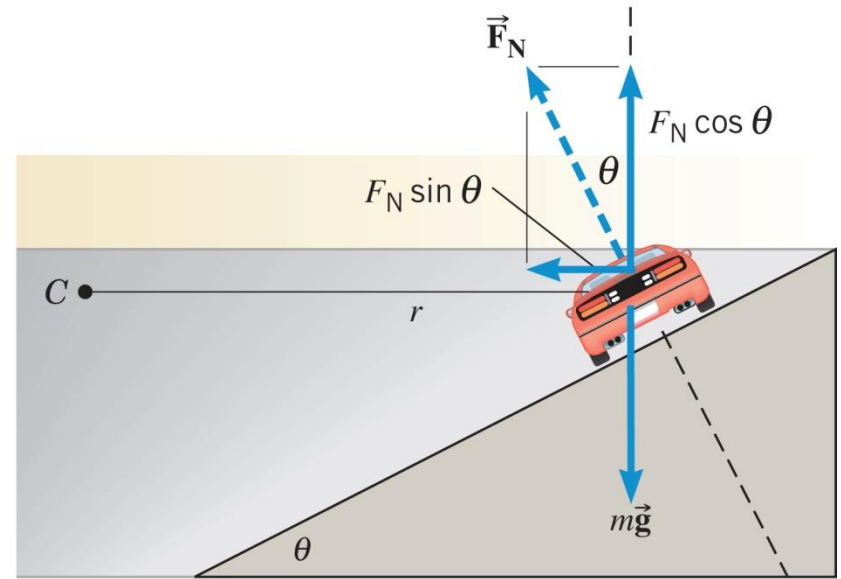


(b)

5.4 Banked Curves



(a)



(b)

$$F_c = F_N \sin \theta = m \frac{v^2}{r}$$

$$F_N \cos \theta = mg$$

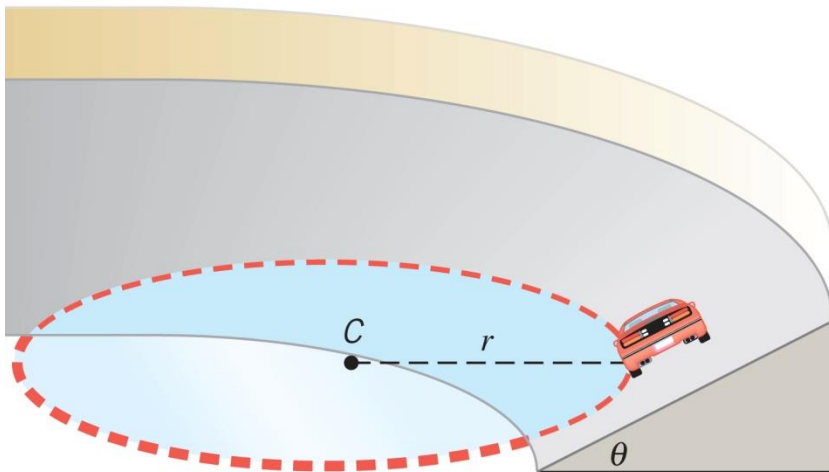
5.4 Banked Curves

$$F_N \sin \theta = m \frac{v^2}{r}$$

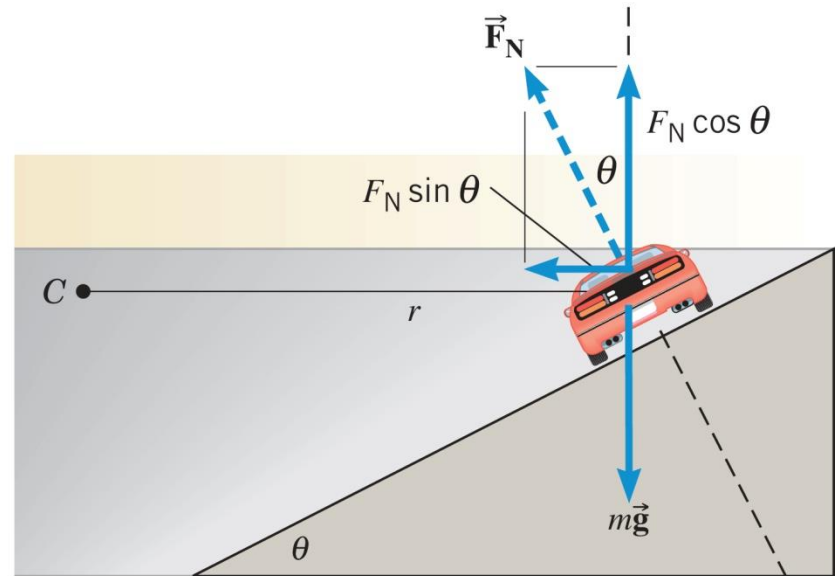


$$\tan \theta = \frac{v^2}{rg}$$

$$F_N \cos \theta = mg$$



(a)



(b)

5.4 Banked Curves

Example 8: The Daytona 500

The turns at the Daytona International Speedway have a maximum radius of 316 m and are steeply banked at 31 degrees. Suppose these turns were frictionless. At what speed would the cars have to travel around them in order to remain on the track?

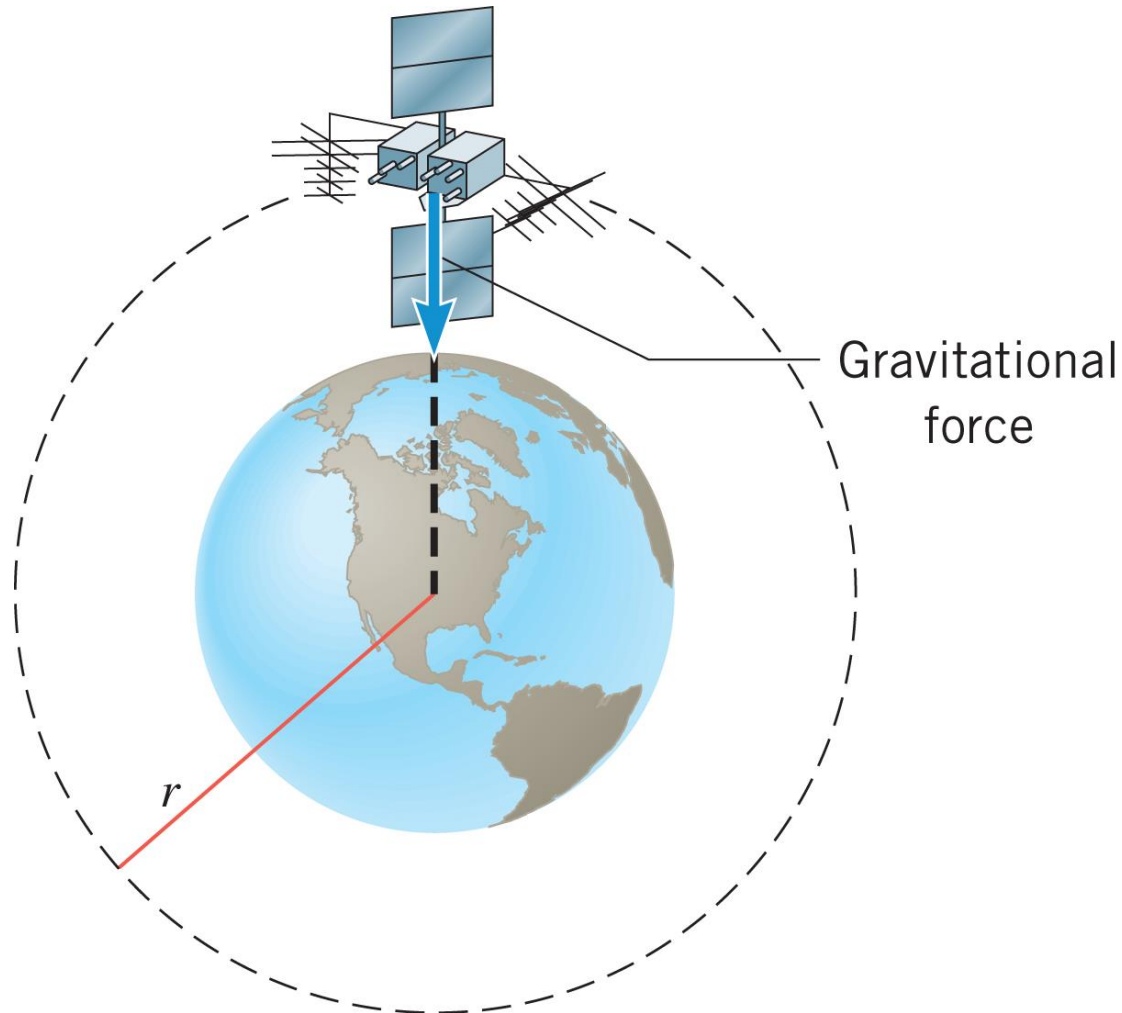
$$\tan \theta = \frac{v^2}{rg} \quad \longrightarrow \quad v = \sqrt{rg \tan \theta}$$

$$v = \sqrt{(316 \text{ m})(9.8 \text{ m/s}^2) \tan 31^\circ}$$

43 m/s (96 MPH)

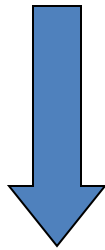
5.5 Satellites in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.

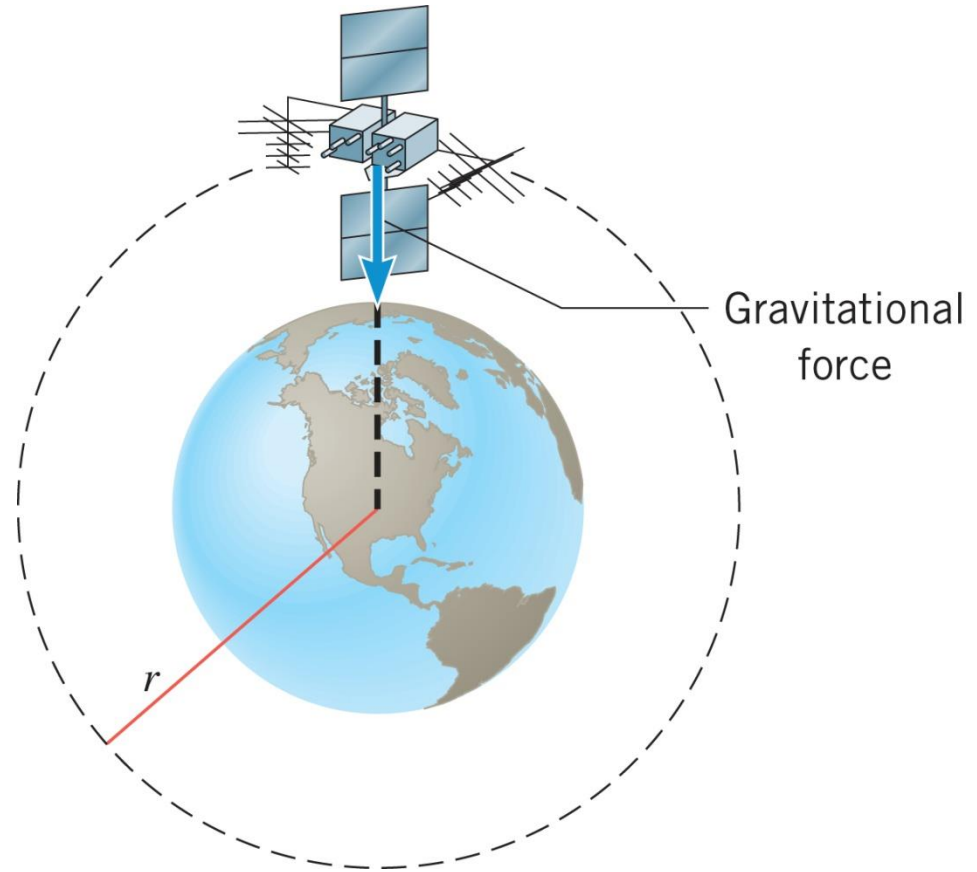


5.5 Satellites in Circular Orbits

$$F_c = G \frac{mM_E}{r^2} = m \frac{v^2}{r}$$



$$v = \sqrt{\frac{GM_E}{r}}$$



Example 9: Orbital Speed of the Hubble Space Telescope

Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.38 \times 10^6 \text{ m} + 598 \times 10^3 \text{ m}}}$$

$$= 7.56 \times 10^3 \text{ m/s} \quad (16900 \text{ mi/h})$$

Period of a Satellite

$$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$



$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

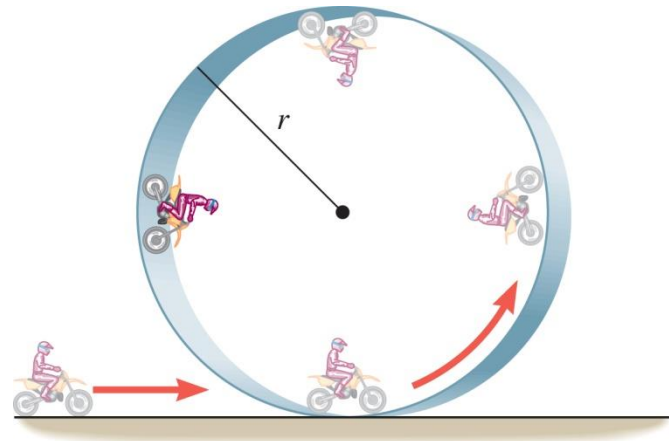
5.7 Vertical Circular Motion

$$F_{N1} - mg = m \frac{v_1^2}{r}$$

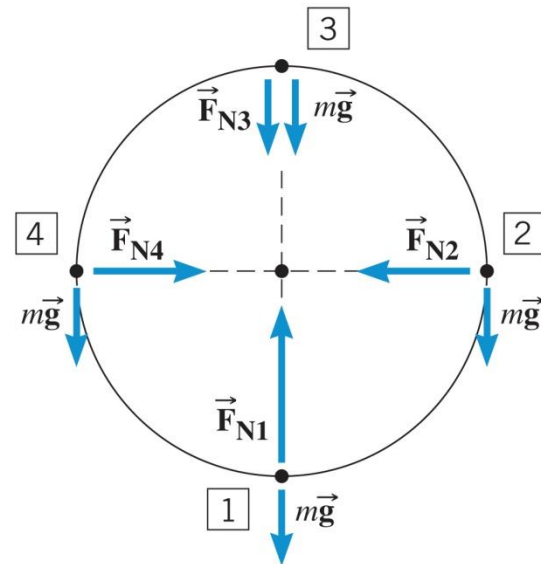
$$F_{N2} = m \frac{v_2^2}{r}$$

$$F_{N3} + mg = m \frac{v_3^2}{r}$$

$$F_{N4} = m \frac{v_4^2}{r}$$



(a)



(b)

For Practice

FOC Questions:

1, 3, 7, 8, 10 and 15

Problems:

4, 17, 23, 25 & 47

Exam-I covering Ch 1 through 5 will be on Sep 18th weekend on D2L